Mathematical methods in physics - course description

General information

Course name	Mathematical methods in physics
Course ID	13.2-WF-FizP-MMP-S17
Faculty	Faculty of Physics and Astronomy
Field of study	Physics
Education profile	academic
Level of studies	First-cycle studies leading to Bachelor's degree
Beginning semester	winter term 2018/2019

Course information

Semester	3
ECTS credits to win	6
Course type	obligatory
Teaching language	english
Author of syllabus	• prof. dr hab. Andrzej Maciejewski

Classes forms

The class form	Hours per semester (full-time)	Hours per week (full-time)	Hours per semester (part-time)	Hours per week (part-time)	Form of assignment
Lecture	30	2	-	-	Exam
Class	30	2	-	-	Credit with grade

Aim of the course

Acquainting the student with advanced mathematical methods necessary for understanding the contents of main study subjects.

Prerequisites

Mathematical analysis I and II together with algebraic and geometric methods in physics.

Scope

- Elements of analytical geometry: planar and space curves, tangents and normals to planar curves, various parameterizations of of straight line, conics in Cartesian and polar coordinates, equations of plane in space, surfaces, quadrics and their classifications.

- Differential operators in curvilinear coordinates: planar and spatial Cartesian and curvilinear coordinates, curvilinear orthogonal coordinates, scalar and vector fields, differential operations on scalar and vector fields: gradient, divergence, rotation, Laplace operator in Cartesian coordinates; potential fields, divergence free fields and irrotational fields; gradient, divergence, rotation, Laplace operator in curvilinear orthogonal coordinates. Definition of tensor fields and algebraic operations on them.

- Elements of variational calculus: definition of functional and examples of them, weak and strong extrema, notion of variation of functional, necessary condition for existence of extremum of a functional, Eulera-Lagrange equations and their properties. Applications of variational calculus.

- Functions of complex variable: complex function of complex variable, limit of function, continuity of function, derivative of complex function, Cauchy-Riemanna conditions for the existence of the complex derivative, Cauchy integral formula, Taylor and Laurent series, singular points of a function, residue, calculation of integrals with the help of residue theory.

- Ordinary differential equations: first order differential equations: method of isoclines, finding solutions of various types of differential equations: separable, homogeneous, Bernoulli's and Riccati's equations, second order linear homogeneous and non-homogeneous differential equations with constant and variable coefficients, method of constant variations and method of undetermined coefficients.

- Partial differential equations of mathematical physics: vibrating string equation and d'Alembert method, membrane equation and Fouriera method of variables separation, Laplace equation.

Teaching methods

Conventional lecture. Computational problems illustrating the lecture material together with its physical applications.

Learning outcomes and methods of theirs verification

Outcome description	Outcome symbols	Methods of verification	The class form
Student knows how to check if a complex function is differentiable and calculates its derivatives, knows parametrisation of the most important curves on complex plane and calculates integrals of complex functions, applies Cauchy integral formula to determine integrals of complex functions. Student knows definition of Taylor series and expands given function into Taylor series, understands the notion of holomorphic function, knows the singular points classification. Student knows the definitions of Laurent series and residue, calculates residua using different methods, applies residues to calculate integrals		• an evaluation test • an exam - oral, descriptive, test and other	• Lecture • Class

Outcome description	Outcome symbols	Methods of verification	The class form
Student knows and uses various parameterizations of planar and spatial curves, can write the straight line equation knowing various sets of given data, determines equations of tangents and normals to given planar curves, recognizes types of conics from their equations, rewrites conics equations from Cartesian to polar coordinates and vice versa, writes conics equations in coordinates frames with shifted origin		• an evaluation test • an exam - oral, descriptive, test and other	• Lecture • Class
Student can solve basic classes of first and second order ordinary differential equations. Student knows fundamental partial differential equations: string, membrane and Laplace equations and knows simplest methods of solving them		• an evaluation test • an exam - oral, descriptive, test and other	• Lecture • Class
Student knows extremum condition for functionals and applies it for various problems of mathematics and physics		• an evaluation test • an exam - oral, descriptive, test and other	• Lecture • Class
Student knows various types of curvilinear coordinates, can check whether the coordinates are orthogonal, determines Lame coefficients, knows how to determine gradient, divergence, rotation and Laplace operator in given orthogonal coordinates.; applies the properties of the Kronecker delta and Levi-Civita’s symbol for derivation of various vectorial identities. Student can check if vector fields are divergence free or irrotational, determines scalar and vectorial potential for given vector fields; can transform scalar functions and vectorial fields from one to another coordinates system.		• an evaluation test • an exam - oral, descriptive, test and other	• Lecture • Class
Student knows various types of curvilinear coordinates, can check whether the coordinates are orthogonal, determines Lame coefficients, knows how to determine gradient, divergence, rotation and Laplace operator in given orthogonal coordinates.; applies the properties of the Kronecker delta and Levi-Civita’s symbol for derivation of various vectorial identities. Student can check if vector fields are divergence free or irrotational, determines scalar and vectorial potential for given vector fields; can transform scalar functions and vectorial fields from one to another coordinates system.		• an evaluation test • an exam - oral, descriptive, test and other	• Lecture • Class
Student knows and understands selected problems from analytical geometry, vector analysis, variational calculus, functions of complex variable and practical aspects of ordinary and partial differential equations of selected types. Student knows elementary terminology employed in these areas of science		• an evaluation test • an exam - oral, descriptive, test and other	• Lecture • Class

Assignment conditions

Lecture: Exam. The course credit is obtained by passing a final written exam composed of tasks of varying degrees of difficulty.

Class: Written test. A student is required to obtain at least the lowest passing grade from the test organized during class.

To be admitted to the exam a student must receive a credit for the class.

Final grade: weighted average of grades from exam (60%) and class (40%).

Recommended reading

[1] R. Leitner, *Zarys matematyki wyższej*, część I, II i III, WNT, Warszawa 1998.

[2] D. McQuarrie, *Matematyka dla przyrodników i inżynierów,* T. 1, 2 i 3, PWN, Warszawa 2006.

[3] T. Jurlewicz, Z. Skoczylas, *Algebra i geometria analityczna*, Oficyna Wydawnicza GiS, Wrocław 2011.

[4] E. Karaśkiewicz, *Zarys teorii wektorów i tensorów*, PWN, Warszawa 1974.

[5] I. M. Gelfand, S. W. Fomin, *Rachunek wariacyjny*, PWN, Warszawa 1970.

[6] J. Długosz, *Funkcje zespolone*, Oficyna Wydawnicza GiS, Wrocław 2005.

[7] M. Gewert, Z. Skoczylas, *Równania różniczkowe zwyczajne*, Oficyna Wydawnicza GiS, Wrocław 2006.

[8] G. I. Zaporożec, *Metody rozwiązywania zadań z analizy matematycznej*, WNT, Warszawa 1976.

Further reading

[1] F. W. Byron, R. W. Fuller, *Metody matematyczne w fizyce klasycznej i kwantowej*, t. 1-2, PWN, Warszawa 1974,

eng. F. W. Byron, R. W. Fuller, Mathematics of Classical and Quantum Physics, vol I and II *Dover Publications, Inc., New York,* 1992

[2] J. Bird, *Higher engineering mathematics*, Elsevier, Amsterdam 2006.

[3] A. Dubrovin, S. P. Novikov, A.T. Fomenko *Modern Geometry. Methods and Applications,* Part 1, Springer-Verlag, 1984.

Notes

Modified by dr hab. Piotr Lubiński, prof. UZ (last modification: 01-08-2018 14:41)