

# Algebra liniowa 1 - opis przedmiotu

## Informacje ogólne

Nazwa przedmiotu	Algebra liniowa 1
Kod przedmiotu	11.1-WK-MATP-AL1-Ć-S14_pNadGenEIR82
Wydział	<a href="#">Wydział Matematyki, Informatyki i Ekonometrii</a>
Kierunek	Mathematics
Profil	ogółnoakademicki
Rodzaj studiów	pierwszego stopnia z tyt. licencjata
Semestr rozpoczęcia	semestr zimowy 2019/2020

## Informacje o przedmiocie

Semestr	1
Liczba punktów ECTS do zdobycia	6
Typ przedmiotu	obowiązkowy
Język nauczania	polski
Syllabus opracował	• dr hab. Krzysztof Przesławski, prof. UZ

## Formy zajęć

Forma zajęć	Liczba godzin w semestrze (stacjonarne)	Liczba godzin w tygodniu (stacjonarne)	Liczba godzin w semestrze (niestacjonarne)	Liczba godzin w tygodniu (niestacjonarne)	Forma zaliczenia
Ćwiczenia	45	3	-	-	Zaliczenie na ocenę
Wykład	45	3	-	-	Egzamin

## Cel przedmiotu

To equip students with knowledge concerning basic algebraic structures such as fields, groups, vector spaces.

## Wymagania wstępne

Secondary school mathematics.

## Zakres tematyczny

### Lecture

#### Fields

1. Number fields. (2h)
2. Operations. Axiomatic definition of a field. (2h)
3. The field of rational functions (1h)
4. The field of residue classes modulo p; Fermat's little theorem (3h)
5. Isomorphisms of fields; automorphisms. The characteristic of a field. (2h)
6. Complex numbers: conjugation, modulus, polar form, geometric interpretation of addition and multiplication, De Moivre's theorem, roots of complex numbers. (4h)
7. Fundamental theorem of algebra. Algebraic and transcendental numbers. (basic information) (1h)
8. Noncommutative fields: quaternions. (only briefly; students have to expand their knowledge by self study)(1h)

#### Permutations

1. Definition of a group; examples. (1h)
2. Parity of a permutation; alternating groups. (2h)
3. Decomposition of a permutation into disjoint cycles; decomposition into transpositions. (1h)

#### Vector spaces

1. Definition of a vector space; examples. (1h)
2. Linear independence; subspaces and spanning sets; basis; the Steinitz exchange lemma; dimension.(2h)
3. Linear transformations; spaces of linear homomorphisms; isomorphisms; linear transformations between coordinate spaces and their matrices; matrix multiplication and composition of linear transformations; algebras over a field: algebras of linear endomorphisms. (3h)
4. Rank of a matrix; the kernel and image of a linear transformation. (3h)
5. Matrix of a linear transformation with respect to arbitrary bases. (2h)
6. Dual space; dual basis; double dual and the canonical isomorphism between a space and its double dual; transpose of a linear transformation; transposed matrix. (2h)

#### Determinants

1. Determinant of a square matrix; multilinearity of determinant. (2h)
2. Determinant of a product of two matrices; determinant of a linear endomorphism. (2h)
3. Laplace expansion; inverse of a matrix. (1h)
4. General linear group, special linear group; group of upper triangular matrices. (1h)

#### Systems of linear equations

1. Existence of solutions (2h)
2. Fundamental system of solutions; dimension of the space of solutions. (2h)

3. Form of the solution to the system  $Ax=b$ , when A is an invertible matrix. (1h)

4. Gauss-Jordan elimination. (1h)

## Class

### Fields

1. Rational and irrational numbers; examples. Number fields; examples. (3h)

2. Two-argument operations and their properties. (1h)

3. Modular computations: tables of operations, inverse elements; binomial coefficients (exercises with the use of mathematical induction); applications of Fermat's little theorem. (2h)

4. Complex numbers: finding products of numbers, and the inverse and the canonical form of a number. (2h)

5. Finding the argument and the modulus of a number. Roots. (2h)

6. Solving equations with complex coefficients. (2h)

7. Class test. (2h)

### Permutations

1. Finding products of permutations. Inverses. Decompositions of permutations into cycles and transpositions. The sign of a permutation. (4h)

### Vector spaces

1. Examples of vector spaces (2h)

2. Verification of linear independency; bases (2h)

3. Calculating values of linear mapping. Finding the kernel and the image of a linear mapping in some simple cases. (4h)

4. An algorithm for finding the rank of a matrix. (2h)

5. Class test. (2h)

### Determinants

1. Applications of  $2 \times 2$  determinants: the area of a parallelogram and a triangle. (2h)

2.  $3 \times 3$  determinants: the volume of a parallelepiped. (2h)

3. Calculating certain determinants of large size. (4h)

### Systems of linear equations

1. Finding the inverse of a matrix. (1h)

2. Checking the consistency of a linear system (2h)

3. Finding a fundamental system of solutions by Gauss-Jordan elimination. (2h)

4. Class test. (2h)

## Metody kształcenia

Traditional lecturing, solving problems under the supervision of the instructor.

## Efekty uczenia się i metody weryfikacji osiągania efektów uczenia się

Opis efektu	Symbol efektów	Metody weryfikacji	Forma zajęć
Student is capable to perform simple operations on elements of fields, as e.g.: finding the inverse of an element; evaluating a power of an element in the field of residues modulo p; evaluating a root of a complex number.	• K_W05	<ul style="list-style-type: none"><li>• egzamin - ustny, opisowy, testowy i inne</li><li>• obserwacja i ocena aktywności na zajęciach</li><li>• test</li></ul>	<ul style="list-style-type: none"><li>• Wykład</li><li>• Ćwiczenia</li></ul>
Student is able to decide whether a given linear system is consistent; can solve linear systems of medium complexity.	• K_U01	<ul style="list-style-type: none"><li>• egzamin - ustny, opisowy, testowy i inne</li><li>• obserwacja i ocena aktywności na zajęciach</li><li>• test</li></ul>	<ul style="list-style-type: none"><li>• Wykład</li><li>• Ćwiczenia</li></ul>
Student uses the notion of a vector space, linear transformation, matrix, and is capable to solve simple problems formulated with the use of these notions. She/ he is able to find the matrix of a linear transformation with respect to given bases.	• K_W04	<ul style="list-style-type: none"><li>• egzamin - ustny, opisowy, testowy i inne</li><li>• obserwacja i ocena aktywności na zajęciach</li><li>• test</li></ul>	<ul style="list-style-type: none"><li>• Wykład</li><li>• Ćwiczenia</li></ul>
Student is able to calculate the sign of a permutation.	• K_U18	<ul style="list-style-type: none"><li>• egzamin - ustny, opisowy, testowy i inne</li><li>• obserwacja i ocena aktywności na zajęciach</li><li>• test</li></ul>	<ul style="list-style-type: none"><li>• Wykład</li><li>• Ćwiczenia</li></ul>
Student is able to check whether a given system of vectors is linearly independent, and to find the rank of a matrix.	• K_W03 • K_U16 • K_U20	<ul style="list-style-type: none"><li>• egzamin - ustny, opisowy, testowy i inne</li><li>• obserwacja i ocena aktywności na zajęciach</li><li>• test</li></ul>	<ul style="list-style-type: none"><li>• Wykład</li><li>• Ćwiczenia</li></ul>

Opis efektu	Symbol efektów	Metody weryfikacji	Forma zajęć
Student knows and understands the notion of a field, also knows basic examples of this structure.	• <a href="#">K_U19</a>	<ul style="list-style-type: none"> <li>egzamin - ustny, opisowy, testowy i inne</li> <li>obserwacja i ocena aktywności na zajęciach</li> <li>test</li> </ul>	<ul style="list-style-type: none"> <li>Wykład</li> <li>Ćwiczenia</li> </ul>
Student knows the proof of Fermat's little theorem.	• <a href="#">K_U18</a>	<ul style="list-style-type: none"> <li>egzamin - ustny, opisowy, testowy i inne</li> <li>obserwacja i ocena aktywności na zajęciach</li> <li>test</li> </ul>	<ul style="list-style-type: none"> <li>Wykład</li> <li>Ćwiczenia</li> </ul>
Student knows the notion of determinant, and is able to calculate a determinant using the Laplace method; knows geometric interpretation of determinant in two and three dimensions.	• <a href="#">K_W03</a> • <a href="#">K_U19</a>	<ul style="list-style-type: none"> <li>egzamin - ustny, opisowy, testowy i inne</li> <li>obserwacja i ocena aktywności na zajęciach</li> <li>test</li> </ul>	<ul style="list-style-type: none"> <li>Wykład</li> <li>Ćwiczenia</li> </ul>

## Warunki zaliczenia

- Preparation of the students and their active participation is assessed during each class by their instructor.
- Class tests with problems of diverse difficulty helping to assess whether a student achieved minimal outcomes.
- Written examination: It consists of around 18 problems. Each problem consists of 2 or 3 statements. To solve a problem, one has only to decide whether the statements are true or false. For some of them, however, explanations are demanded.

Final grade =  $0.4 \times \text{class grade} + 0.6 \times \text{exam grade}$ . In order to be allowed to take the exam a student has to have a positive class grade. In order to pass the exam a student has to have a positive exam grade.

## Literatura podstawowa

- Strang, Gilbert, Linear Algebra and Its Applications, Cengage Learning, 2005.

## Literatura uzupełniająca

- G. Birkhoff, S. Mac Lane, A Survey of Modern Algebra, A.K. Peters, 1997.

## Uwagi

Zmodyfikowane przez dr Robert Dylewski, prof. UZ (ostatnia modyfikacja: 19-09-2019 13:21)

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