Mathematical methods in physics - course description

General information

Course name	Mathematical methods in physics
Course ID	13.2-WF-FizD-MMP-S17
Faculty	Faculty of Physics and Astronomy
Field of study	Physics
Education profile	academic
Level of studies	Second-cycle studies leading to MS degree
Beginning semester	winter term 2020/2021

Course information

Semester	1
ECTS credits to win	6
Available in specialities	Theoretical physics
Course type	obligatory
Teaching language	english
Author of syllabus	• dr hab. Maria Przybylska, prof. UZ

Classes forms

The class form	Hours per semester (full-time)	Hours per week (full-time)	Hours per semester (part-time)	Hours per week (part-time)	Form of assignment
Lecture	30	2	-	-	Exam
Laboratory	30	2	-	-	Credit with grade

Aim of the course

To teach the students basic mathematical tools of differential geometry and tensor analysis necessary to study general relativity.

Prerequisites

Mathematical analysis I and II, and algebraic and geometric methods in physics.

Scope

- Elements of multivariable functions analysis: functions from R^n to R^m, continuity, limits, differentiability, Jacobi matrix of transformation, inverse and implicit function theorems.
- Elements of differential geometry: Cartesian and curvilinear coordinate systems, in R^n and in a domain of R^n, Curves in Euclidean space, length of curve, Riemannian metrics, natural parametrisation of curve, curvature and torsions, Serret-Frenet formulae, surfaces in R^3, first and second fundamental form of surfaces, mean and Gauss curvatures, hypersurfaces immersed in higher-dimensional flat spaces, notion of differential manifold, coordinates on differential manifold, tangent and cotangent spaces.
- Elements of tensor algebra. Space dual to a vector space, multilinear mapping, transformation laws for tensor and tensor fields, algebraic operations on tensors, differential forms as skew-symmetric tensors, examples of applications of tensors in physics.
- Elements of tensor analysis: affine connection, covariant derivative, Christoffel symbols, torsion, Riemannian connection, parallel displacement, equation of parallel displacement, geodesics, curvature tensor, Euclidean coordinate, properties of the Riemann curvature tensor, curvature scalar.

Teaching methods

Conventional lecture with emphasis on contents useful for studies of general relativity
During class students solve exercises illustrating the content of the lecture with examples related to general relativity

Learning outcomes and methods of theirs verification

Outcome description	Outcome symbols	Methods of verification	The class form

Outcome description	Outcome symbols	Methods of verification	The class form
The student knows and understands selected issues multivariate analysis, differential geometry and tensor algebra and analysis. He is familiar with the terminology used in these sciences.	• K2_W02	• a pass - oral, descriptive, test and other • an exam - oral, descriptive, test and other	• Lecture • Laboratory
Student is aware of his knowledge and skills. Student recognise the necessity of permanent training and improvement of his knowledge from application of mathematics to general relativity as well as to contemporary physics.	• K2_K01	• a discussion	• Laboratory
Student can find on their own various teaching materials concerning differential geometry and tensor calculus in Polish and English.	• K2_U09	• a pass - oral, descriptive, test and other • an exam - oral, descriptive, test and other	• Lecture • Laboratory
Student can use mathematical methods to describe and model physical phenomena and processes.	• K2_W05	• a discussion • a pass - oral, descriptive, test and other • an exam - oral, descriptive, test and other	• Lecture • Laboratory
Student knows and applies various curvilinear coordinates, determines domain of their definiteness, Student determines natural parametrisation of given curves, calculates curvatures and torsions of curves. Student calculates fundamental forms and curvatures of surfaces.	• K2_W02 • K2_W05 • K2_U05	• a pass - oral, descriptive, test and other • an exam - oral, descriptive, test and other	• Lecture • Laboratory
Student can transform of tensor fields of various types under change of coordinates, make algebraic operations of tensors, calculate Christoffel symbols from metrics and from geodesic equations, determines geodesics. Student calculates curvature tensor and curvature scalar, knows properties of curvature tensor and apply them.	• K2_W02 • K2_W05 • K2_U05	• a pass - oral, descriptive, test and other • an exam - oral, descriptive, test and other	• Lecture • Laboratory

Assignment conditions

Lecture:
The course credit is obtained by passing a final written exam composed of tasks of varying degrees of difficulty.
Class:
A student is required to obtain at least the lowest passing grade from tests organized during class.
To be admitted to the exam a student must receive a credit for the class
Final grade: average of grades from the class and the exam.

Recommended reading

[1] L. M. Sokołowski, Elementy analizy tensorowej, Wydawnictwo Uniwersytetu Warszawskiego, 2010.

[2] M. Spivak, Analiza na rozmaitościach, Wydawnictwo Naukowe PWN, Warszawa 2006.

[3] A. Goetz i inni, Zewnętrzne formy różniczkowe, WNT, Warszawa 1965.

[4] S. Lovett, Differential geometry of Manifolds, A K Peters, Ltd, Natick, Massachusetts 2010.

[5] A. S. Mishchenko, A. Fomenko, A course of Differential Geometry and Topology, Mir Publishers Moscow 1988.

[6] B. A. Dubrovin, A.T. Fomenko, S.P. Novikov, Modern Geometry – Methods and Applications,

Springer 1992.

[7] A. S. Mishchenko, Yu. P. Solovyev,, A.T. Fomenko, Problems in Differential Geometry and

Topology, Mir Publishers, Moscow 1985.

Further reading

[1] P. M. Gadea, J. Munoz Masque, Analysis and Algebra on Differentiable Manifolds, Springer, 2009.
[2] T. Banchoff, S. Lovett, Differential Geometry of Curves and Surfaces, A K Peters, Ltd, Natick, Massachusetts 2010.
[3] S. Chandrasekhar, The Mathematical Theory of Black Holes, Clarendon Press, Oxford 1983.
[4] E. Karaśkiewicz, Zarys teorii wektorów i tensorów, Państwowe Wydawnictwo Naukowe, Warszawa 1964.

Notes

Modified by dr hab. Piotr Lubiński, prof. UZ (last modification: 09-06-2020 17:03)