Linear Algebra 2 - course description

General information

Course name	Linear Algebra 2
Course ID	11.1-WK-MATP-AL2-Ć-S14_pNadGenINHLH
Faculty	Faculty of Mathematics, Computer Science and Econometrics
Field of study	Mathematics
Education profile	academic
Level of studies	First-cycle studies leading to Bachelor's degree
Beginning semester	winter term 2020/2021

Course information

Semester	2
ECTS credits to win	6
Course type	obligatory
Teaching language	polish
Author of syllabus	• dr hab. Krzysztof Przesławski, prof. UZ

Classes forms

The class form	Hours per semester (full-time)	Hours per week (full-time)	Hours per semester (part-time)	Hours per week (part-time)	Form of assignment
Class	30	2	-	-	Credit with grade
Lecture	30	2	-	-	Exam

Aim of the course

The objective of the whole course (linear algebra 1 and 2) is to prepare participants to self-study of theoretical and practical problems involving methods of linear algebra. The aim of each student should be to master the material included in the recommended book.

Prerequisites

Linear algebra 1.

Scope

Lecture

Systems of linear equations

1. Characteristic equation; eigenvectors; eigenvalues; examples and applications. (4h)

Jordan decomposition

1. Algebraic sum of linear subspaces; direct sum. (1h)
2. Nilpotent endomorphisms; Jordan blocks. Invariant subspaces of an endomorphism. (2h)
3. Jordan decomposition of an endomorphism; Jordan normal form. (2h)

Euclidean spaces

1. Cosine theorem — geometric definition of a scalar product; scalar product in coordinate spaces. (1h)
2. Formal definition of a scalar product; norm; Schwarz inequality; angle between two vectors, triangle inequality; parallelogram law. (2h)
3. Orthogonality: Pythagorean theorem, orthonormal basis.(1h)
4. Gram–Schmidt algorithm, existence of an orthonormal basis, expansion of a vector with respect to an orthogonal basis, orthogonal complement. (3h)
5. Isomorphic Euclidean spaces; canonical isomorphism between a Euclidean space and its dual. (1h)
6. Conjugate of a linear transformation; spectral theorem for self-adjoint operations.

7. Orthogonal transformations; decomposition of a space into minimal invariant subspaces: rotations, reflections. Canonical matrix of an orthogonal transformation. Orientation.(5h)

Bilinear forms

1. Multilinear forms: skew forms, symmetric forms. (1h)
2. Bilinear symmetric forms: matrix of a form with respect to a given frame. (1h)
3. Diagonalization of a bilinear symmetric form; Sylvester's law. (2h)
4. Quadratic forms; polarization formula — the one-to-one correspondence between symmetric and quadratic forms. (1h)

Class

Systems of linear equations

1. Solving eigenvalue problems. (4h)

Jordan decomposition

1. Simple examples. Information on numerical packages. (2h)

Euclidean spaces

1. Finding the angle between vectors. Checking whether a given form is a scalar product (2h)
2. Finding an orthonormal basis by Gram–Schmidt orthogonalisation process. Gram's determinant and its geometrical interpretation. (5h)
3. Class test

4. Diagonalisation of simple self-adjoint transformations. (4h)
5. Classification of orthogonal transformations in dimensions 2 and 3. Composition of orthogonal transformations. Reduction of orthogonal matrices to their canonical forms – examples. (5h)
Bilinear forms
1. Matrix of a bilinear form. Decompsition of a form into skew and symmetric parts. (1h)
2. Diagonalization of bilinear forms (quadratic forms). (2h)

Teaching methods

Traditional lecturing, solving problems under the supervision of the instructor.

Learning outcomes and methods of theirs verification

Outcome description	Outcome symbols	Methods of verification	The class form
Student understands the meaning of an abstract Euclidean space for geometrisation of practical problems; is able to calculate an appropriate orthonormal basis; knows, and is able to find the Fourier expansion of a vector; can find the basis of eigenvectors for a simple self-adjoint transformation.	• K_U20	• a test • an exam - oral, descriptive, test and other • an observation and evaluation of activities during the classes	• Lecture • Class
Student knows, on an operational level, basic theorems of linear algebra.	• K_W07 • K_U16 • K_U21	• a test • an exam - oral, descriptive, test and other • an observation and evaluation of activities during the classes	• Lecture • Class
Student is capable to reduce an orthogonal transformation to its canonical form in simple two or three dimensional cases; knows how to reduce a quadratic form to its canonical form.	• K_U21	• a test • an exam - oral, descriptive, test and other • an observation and evaluation of activities during the classes	• Lecture • Class
Student knows the notion of an eigenvalue, and an eigenvector. He is able to find them for problems of medium complexity (e.g. small size; presence of symmetries).	• K_W03 • K_W04	• a test • an exam - oral, descriptive, test and other • an observation and evaluation of activities during the classes	• Lecture • Class

Assignment conditions

1. Preparation of the students and their active participation is assessed during each class by their instructor.
2. Class tests with problems of diverse difficulty helping to assess whether a student achieved minimal outcomes.
3. Written examination: It consists of around 18 problems. Each problem consists of 2 or 3 statements. To solve a problem, one has only to decide whether the statements are true or false. For some of them, however, explanations are demanded.

Final grade = 0.4 x class grade + 0,6 x exam grade. In order to be allowed to take the exam a student has to have a positive class grade. In order to pass the exam a student has to have a positive exam grade

Recommended reading

1. Strang, Gilbert, Linear Algebra and Its Applications, Cengage Learning, 2005.

Further reading

1. G. Birkhoff, S. Mac Lane, A Survey of Modern Algebra, A.K. Peters, 1997.

Notes

Modified by dr Alina Szelecka (last modification: 18-09-2020 13:45)