Mathematical Analysis 2 - course description

General information

Course name	Mathematical Analysis 2
Course ID	11.1-WK-MATP-AM2-W-S14_pNadGen1OVIJ
Faculty	Faculty of Mathematics, Computer Science and Econometrics
Field of study	Mathematics
Education profile	academic
Level of studies	First-cycle studies leading to Bachelor's degree
Beginning semester	winter term 2020/2021

Course information

Semester	2
ECTS credits to win	10
Course type	obligatory
Teaching language	polish
Author of syllabus	• prof. dr hab. Witold Jarczyk • prof. dr hab. Janusz Matkowski

Classes forms

The class form	Hours per semester (full-time)	Hours per week (full-time)	Hours per semester (part-time)	Hours per week (part-time)	Form of assignment
Lecture	60	4	-	-	Exam
Class	60	4	-	-	Credit with grade

Aim of the course

To acquaint students with differential methods of examining extrema and the convexity of a function, with the notions of the primitive function and Riemann integral. The emphasis is placed on mastering calculating techniques, in particular those of integrating, and also on applications of differential and integral calculus. The next aim is to transfer basics of differential calculus on functions in several variables.

Prerequisites

Mathematical Analysis 1. Logic and Set Theory. Linear Algebra 1.

Scope

Lecture

I. Elementary differential calculus II

1. Local extrema (1 hour)
2. Characterization of the convexity of a function (1 hour)
3. Relationships of the uniform convergence to differentiating (2 hours)
4. Differentiability of elementary functions (1 hour)
5. Primitive function (2 hours)
6. Algorithm of integrating rational functions (the material should be prepared in student's own right basing on a literature indicated by the lecturer)
7. Derivative of a function of a convex variable (a brief information) (1 hour)

II. Applications of differential calculus (the material should be elaborated in a written form by teams of students basing on a literature indicated by the lecturer)

1. Straight-line motion.
2. Applications to geometry.
3. Differential and approximate calculation.
4. Newton method.
5. Applications in economics.

III. Elementary integral calculus

1. Riemann integral and area. Basic properties of integral. Mean value theorem for integrals (8 hours)
2. Relationships of differentiation to integration. Newton-Leibniz fundamental theorem of calculus and its consequences (3 hours)
3. Relationships of uniform convergence to integration. Integrating series of functions (2 hours)
4. Improper integral (4 hours)

IV. Techniques of integration

1. Trigonometric substitutions (2 hours)
2. Euler's substitutions (2 hours)
3. Numerical integration: trapezoidal rule, Simpson's rule (the material should be prepared in student's own right basing on a literature indicated by the lecturer)

V. Applications of integral calculus

1. Exemplary applications of integration in geometry: areas of regions in the plane, volumes of solids, area of surfaces (2 hours)
2. Center of mass and moments. Theorems of Pappus (the material should be prepared in student's own right basing on a literature indicated by the lecturer)
3. Work and pressure (the material should be prepared in student's own right basing on a literature indicated by the lecturer)

VI. Polar coordinates and parametric equations

1. Polar coordinate system. Curves in polar coordinates. Area of a region bounded by a curve. Length of a curve (3 hours)
2. Parametric equations of a curve on the plane. Tangent line to a curve. Length of a curve (2 hours)

VII. Cartesian spaces
1. Scalars and vectors (1 hour)
2. Cylindrical coordinates and spherical coordinates (1 hour)
VIII. Functions of several variables
1. Level sets of functions of two or three variables (1 hour)
2. Limit and continuity (5 hours)
IX. Differential calculus of functions of several variables I
1. Directional and partial derivatives. Jacobi matrix and gradient (2 hours)
2. Differential and differentiability (7 hours)
3. Geometric interpretation of differentiability. Tangent plane and normal line (2 hours)
4. Regular mappings and diffeomorphisms (2 hours)
5. Implicit function theorem (3 hours)
Class
I. Elementary differential calculus II
1. Determination of local and global extrema. Proving inequalities by finding extrema. Function analysis (6 hours)
2. Examining the uniform convergence of sequences of functions and series of functions (2 hours)
3. Taylor's expansion of a function (4 hours)

III. Elementary integral calculus, IV. Techniques of integration and V. Applications of integral calculus
1. Calculating integrals by using definition (2 hours)
2. Integrating by parts and by substitution. Algorithm of integrating rational functions. Making use of Newton-Leibniz fundamental theorem of calculus (10 hours)
Colloquium (2 hours)

3. Convergence and integration. Integrating series of functions (2 hours)
4. Calculating areas of regions in the plane and volumes of solids (3 hours)
5. Determination of the center of mass and calculating the quantity of work (1 hour)
VI. Polar coordinates and parametric equations
1. Changing Cartesian coordinates into polar ones and conversely (2 hours)
2. Calculating areas of regions and length of curves described by polar equations (2 hours)
3. Determination of lines tangent to a curve described parametrically. Calculating areas of regions and length of curves described parametrically (3 hours)
VII. Cartesian spaces
1. Describing surfaces in spherical and cylindrical coordinates (1 hour)
Colloquium (2 hours)
VIII. Functions of several variables
1. Limits and continuity. Iterated limits. Continuity in separated variables (3 hours)
IX. Differential calculus of functions of several variables I
1. Finding directional derivatives, derivative and differential (5 hours)
2. Determination of tangent and normal lines and planes (2 hours)
3. Examining regularity and diffeomorphicity of mappings (3 hours)
4. Studying the problem of implicit functions (3 hours)
Colloquium (2 hours)

Teaching methods

Traditional lecture; class where students, leaded by the teacher, solve exercises and discuss; team-work completed with a written composition; work over a book; making use of internet.

Learning outcomes and methods of theirs verification

Outcome description	Outcome symbols	Methods of verification	The class form
Student learns the notion of Riemann integral and its interpretation and the algorithm of integrating rational functions; knows basic methods of integration.	• K_W04	• a check work • an exam - oral, descriptive, test and other • an observation and evaluation of activities during the classes	• Lecture • Class
Student knows simple examples of applications of differential calculus.	• K_W05	• a check work • an exam - oral, descriptive, test and other • an observation and evaluation of activities during the classes	• Lecture • Class

Outcome description	Outcome symbols	Methods of verification	The class form
Student knows methods of mathematical analysis helpful while constructing models of medium complexity outside mathematics.	• K_W07	• a check work • an exam - oral, descriptive, test and other • an observation and evaluation of activities during the classes	• Lecture • Class
Student knows necessary and sufficient conditions of the existence of local extrema of a differential function.	• K_W04 • K_W07	• a check work • an exam - oral, descriptive, test and other • an observation and evaluation of activities during the classes	• Lecture • Class
Student finds partial derivatives and differentials, determines tangent and normal lines and planes; can decide if a given mapping is a diffeomorphism.	• K_W03	• a check work • an exam - oral, descriptive, test and other • an observation and evaluation of activities during the classes	• Lecture • Class
Student understands the proof of Newton-Leibniz fundamental theorem of calculus and is aware of consequences of that theorem; realizes basic notions and results of differential calculus of functions in several variables; wises and understand implicit function theorem.	• K_U12	• a check work • an exam - oral, descriptive, test and other • an observation and evaluation of activities during the classes	• Lecture • Class
Student is able single-handedly to seek out information in literature and internet; realizes the need of continued education.	• K_U14	• an observation and evaluation of activities during the classes	• Lecture • Class
Student can change the Cartesian coordinates into polar ones and vice versa.	• K_U11	• a check work • an exam - oral, descriptive, test and other • an observation and evaluation of activities during the classes	• Lecture • Class
Student makes use of various techiques of integration and can apply integration to calculating areas of regions, volumes of solids, length of curves.	• K_U12	• a check work • an exam - oral, descriptive, test and other • an observation and evaluation of activities during the classes	• Lecture • Class
Student can make the function analysis and give Taylor's expansion of basic functions; can decide if a given mapping is a diffeomorphism.	• K_U06 • K_K01	• a check work • an exam - oral, descriptive, test and other • an observation and evaluation of activities during the classes	• Lecture • Class

Assignment conditions

1. Verifying the extent of preparation of students and their activity during the classes.
2. Three colloquia with problems of various degree of difficulties, allowing to verify if students attained learning outcomes at the very least.
3. Written compositions elaborated a material indicated by the lecturer and prepared by teams of students.
4. Exam (writ) with indicated point ranges.

The final grade is the arithmetic mean of those of the class and exam. A necessary condition to enter the exam is a positive grade of the classes. A necessary condition to pass the course is a positive grade of the exam.

Recommended reading

1. Witold Jarczyk, Notatki do wykładu z analizy matematycznej, http://www.wmie.uz.zgora.pl/~`wjarczyk/materialy.html
2. Witold Jarczyk, Zadania z analizy matematycznej, http://www.wmie.uz.zgora.pl/~`wjarczyk/materialy.html
3. J. Douglas Faires, Barbara T. Faires, Calculus, Random House, New York

Further reading

1. Józef Banaś, Stanisław Wędrychowicz, Zbiór zadań z analizy matematycznej, Wydawnictwo Naukowo-Techniczne, Warszawa, 1993.
2. Andrzej Birkholc, Analiza matematyczna. Funkcje wielu zmiennych, Wydawnictwo Naukowe PWN, Warszawa, 2002.
3. Witold Kołodziej, Analiza matematyczna, Państwowe Wydawnictwo Naukowe, Warszawa, 1986.
4. Walter Rudin, Podstawy analizy matematycznej, Wydawnictwo Naukowe PWN, Warszawa, 2002.

Notes

Modified by dr Alina Szelecka (last modification: 18-09-2020 13:45)