

# Probability Theory - opis przedmiotu

## Informacje ogólne

Nazwa przedmiotu	Probability Theory
Kod przedmiotu	11.1-WK-MATP-PT-S22
Wydział	Wydział Matematyki, Informatyki i Ekonometrii
Kierunek	WMIiE - oferta ERASMUS
Profil	-
Rodzaj studiów	Program Erasmus
Semestr rozpoczęcia	semestr zimowy 2022/2023

## Informacje o przedmiocie

Semestr	1
Liczba punktów ECTS do zdobycia	5
Typ przedmiotu	obieralny
Język nauczania	angielski
Syllabus opracował	• dr Marta Borowiecka-Olszewska

## Formy zajęć

Forma zajęć	Liczba godzin w semestrze (stacjonarne)	Liczba godzin w tygodniu (stacjonarne)	Liczba godzin w semestrze (niestacjonarne)	Liczba godzin w tygodniu (niestacjonarne)	Forma zaliczenia
Wykład	30	2	-	-	Egzamin
Ćwiczenia	30	2	-	-	Zaliczenie na ocenę

## Cel przedmiotu

Familiarizing students with the basic concepts, theorems and methods of reasoning related to the probability theory.

## Wymagania wstępne

Getting a pass in Mathematical Analysis 1 and 2.

## Zakres tematyczny

### Lecture

#### 1. Events and the probability

- The revision of combinatorics. The classical definition of the probability. (2 hrs.)
- The general definition of the probability. The definition and examples of the probability space and the event. Basic properties of the probability. (3 hrs.)
- The geometrical probability. The conditional probability, the law of total probability and Bayes' rule. (3 hrs.)
- The independence of events. The Bernoulli scheme, the most likely number of successes in the Bernoulli scheme. (2 hrs.)

#### 2. Random variables and their distributions

- The definition, examples and properties of the random variable. The distribution of the random variable. The cumulative distribution function of the random variable and its properties. The cumulative distribution function and types of distributions. (4 hrs.)
- Absolutely continuous and discrete distributions. The probability density function and its property. Overview of the most important absolutely continuous and discrete distributions. Mixed distributions. The independence of random variables. (4 hrs.)
- Multidimensional random variables. The joint and marginal distributions, multidimensional and marginal cumulative distribution functions, marginal probability density functions. Connections with independent random variables. Distributions of sums of independent random variables. (3 hrs.)

#### 3. The expectation and moments of random variables

- The expectation and moments of a random variable. Examples of basic absolutely continuous and discrete distributions. The expectation and moments of random variables of mixed distribution, basic properties and interpretations. The variance and the standard deviation of random variables, basic properties and interpretation. (4 hrs.)
- The concept of the covariance and the correlation coefficient of random variables, their connections with independent random variables. Parameters of random vectors. The multidimensional normal distribution. (2 hrs.)

#### 4. Limit theorems

- Chebyshev's inequality, the weak and strong law of large numbers, the central limit theorem and their applications. (3 hrs.)

### Class

#### 1. Events and the probability

- The binomial coefficient and its interpretation. The use of basic combinatorial schemes to exercises related to the classical definition of the probability. (4 hrs.)
- Determination of elementary events and events. Basic properties of the probability. (2 hrs.)

- Exercises that use the geometric probability, the conditional probability, the law of total probability and Bayes' rule. (2 hrs.)
- Checking the independence of events. The calculation of probabilities of events with the assumption of independence. Exercises that use the Bernoulli scheme. (2 hrs.)
- Colloquium (2 hrs.)

## 2. Random variables and their distributions, the expectation and moments of random variables

- The verification whether some functions are random variables, cumulative distribution functions of some random variables. The determination of the cumulative distribution function of a random variable. The analysis of the distribution of a random variable on the basis of the cumulative distribution function. The verification whether some functions are probability density functions. The application of different types of discrete and absolutely continuous distributions in mathematical models. The application of normal distribution in exercises. (7 hrs.)
- The determination of the joint and marginal distributions of two-dimensional random vectors using the tabular method. The determination of two-dimensional and marginal cumulative distribution functions, marginal probability density functions. The verification of the independence of random variables. Distributions of sums of independent random variables. (3 hrs.)
- The determination of the expectation, moments and the variance of random variables. The properties of the expectation and the variance. The application in exercises. Calculations of the covariance and the correlation coefficient of random variables and their connections with the independence. The parameters of two-dimensional random vectors and two-dimensional normal distribution. (4 hrs.)

## 3. Limit theorems

- The application of Chebyshev's inequality to estimate the probability of random variables. The application of the law of large numbers and the central limit theorem in exercises. (2 hrs.)
- Colloquium (2 hrs.)

## Metody kształcenia

A traditional lecture. Solving previously given tasks (exercises and short proofs) during the classes.

## Efekty uczenia się i metody weryfikacji osiągania efektów uczenia się

Opis efektu	Symbol efektów	Metody weryfikacji	Forma zajęć
The student is able to calculate the probability of events, the expectation and the variance of random variables. He knows and is able to use limit theorems to estimate probabilities.		<ul style="list-style-type: none"> <li>• egzamin - ustny, opisowy, testowy i inne</li> <li>• obserwacja i ocena aktywności na zajęciach</li> <li>• praca kontrolna</li> </ul>	<ul style="list-style-type: none"> <li>• Wykład</li> <li>• Ćwiczenia</li> </ul>
The student is able to explain concepts and give examples of the elementary event, the event, the probability measure, the probability space and the random variable.		<ul style="list-style-type: none"> <li>• egzamin - ustny, opisowy, testowy i inne</li> <li>• obserwacja i ocena aktywności na zajęciach</li> <li>• praca kontrolna</li> </ul>	<ul style="list-style-type: none"> <li>• Wykład</li> <li>• Ćwiczenia</li> </ul>
The student is able to analyse the distribution of random variables on the basis of e.g. a cumulative distribution function or a probability density function. He is able to apply different types of discrete and absolutely continuous distributions in mathematical models.		<ul style="list-style-type: none"> <li>• egzamin - ustny, opisowy, testowy i inne</li> <li>• obserwacja i ocena aktywności na zajęciach</li> <li>• praca kontrolna</li> </ul>	<ul style="list-style-type: none"> <li>• Wykład</li> <li>• Ćwiczenia</li> </ul>
The student is able to use the conditional probability, the law of total probability and Bayes' rule. He is able to check the independence of events and use the Bernoulli scheme.		<ul style="list-style-type: none"> <li>• egzamin - ustny, opisowy, testowy i inne</li> <li>• obserwacja i ocena aktywności na zajęciach</li> <li>• praca kontrolna</li> </ul>	<ul style="list-style-type: none"> <li>• Wykład</li> <li>• Ćwiczenia</li> </ul>

## Warunki zaliczenia

1. Checking the level of preparation of students and their activity during the classes.
2. Two colloquia with tasks of varying difficulty which allow to assess whether students have reached a minimum level of learning outcomes.
3. The exam in the form of a multiple-choice test, consisting of several dozen statements that require the verification on the basis of the acquired knowledge. The verification of statements is connected with the use of the theory or making simple calculations. The possible answers are Yes or No. The student may receive +1,-1 or 0 points for each statement.

To pass the class it is necessary to get passing scores in two colloquia. To take the exam it is necessary to pass the class. In order to pass the course it is necessary to get passing score in the exam. The final course grade is based on graded components: the class grade – 50% and the exam grade – 50%.

## Literatura podstawowa

1. G. Grimmett, D. Welsh, Probability: an Introduction (Oxford University Press, 1986).
2. Ch.M. Grinstead, J.L. Snell, Introduction to Probability (American Mathematical Society, 2006).

## Literatura uzupełniająca

1. G. Roussas, Introduction to Probability (Elsevier, 2007).
2. S.M. Ross, Introduction to Probability Models (first two chapters) (Elsevier, 2007).

## Uwagi

Zmodyfikowane przez dr Marta Borowiecka-Olszewska (ostatnia modyfikacja: 13-04-2022 14:54)

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